High School Algebra

Chapter 9 : Quadratics

Quadratics - Solving with Radicals

Objective: Solve equations with radicals and check for extraneous solutions.

Here we look at equations that have roots in the problem. As you might expect, to clear a root we can raise both sides to an exponent. So to clear a square root we can rise both sides to the second power. To clear a cubed root we can raise both sides to a third power. There is one catch to solving a problem with roots in it, sometimes we end up with solutions that do not actually work in the equation. This will only happen if the index on the root is even, and it will not happen all the time. So for these problems it will be required that we check our answer in the original problem. If a value does not work it is called an extraneous solution and not included in the final solution.

When solving a radical problem with an even index: check answers!

Example 442.

$$\begin{array}{rl}
\sqrt{7x+2}=4 & \text{Even index! We will have to check answers} \\
(\sqrt{7x+2})^2=4^2 & \text{Square both sides, simplify exponents} \\
7x+2=16 & \text{Solve} \\
\underline{-2\quad -2} & \text{Subtract 2 from both sides} \\
7x=14 & \text{Divide both sides by 7} \\
\overline{7}\quad \overline{7} & \\
x=2 & \text{Need to check answer in original problem} \\
\sqrt{7(2)+2}=4 & \text{Multiply} \\
\sqrt{14+2}=4 & \text{Add} \\
\sqrt{16}=4 & \text{Square root} \\
4=4 & \text{True! It works!} \\
x=2 & \text{Our Solution}
\end{array}$$

Sample PREVIEW

Example 443.

$$\begin{array}{rl}
\sqrt[3]{x-1}=-4 & \text{Odd index, we don't need to check answer} \\
(\sqrt[3]{x-1})^3=(-4)^3 & \text{Cube both sides, simplify exponents} \\
x-1=-64 & \text{Solve}
\end{array}$$

$$
\begin{array}{rl}
\underline{+1 \quad +1} & \text{Add 1 to both sides} \\
x = -63 & \text{Our Solution}
\end{array}
$$

Example 444.

$$
\begin{array}{rl}
\sqrt[4]{3x+6} = -3 & \text{Even index! We will have to check answers} \\
(\sqrt[4]{3x+6}) = (-3)^4 & \text{Rise both sides to fourth power} \\
3x+6 = 81 & \text{Solve} \\
\underline{-6 \quad -6} & \text{Subtract 6 from both sides} \\
3x = 75 & \text{Divide both sides by 3} \\
\overline{3 \quad\;\; 3} & \\
x = 25 & \text{Need to check answer in original problem} \\
\sqrt[4]{3(25)+6} = -3 & \text{Multiply} \\
\sqrt[4]{75+6} = -3 & \text{Add} \\
\sqrt[4]{81} = -3 & \text{Take root} \\
3 = -3 & \text{False, extraneous solution} \\
\text{No Solution} & \text{Our Solution}
\end{array}
$$

If the radical is not alone on one side of the equation we will have to solve for the radical before we raise it to an exponent

Example 445.

$$
\begin{array}{rl}
x + \sqrt{4x+1} = 5 & \text{Even index! We will have to check solutions} \\
\underline{-x \qquad\qquad -x} & \text{Isolate radical by subtracting } x \text{ from both sides} \\
\sqrt{4x+1} = 5 - x & \text{Square both sides} \\
(\sqrt{4x+1})^2 = (5-x)^2 & \text{Evaluate exponents, recal } (a-b)^2 = a^2 - 2ab + b^2 \\
4x + 1 = 25 - 10x + x^2 & \text{Re} - \text{order terms} \\
4x + 1 = x^2 - 10x + 25 & \text{Make equation equal zero} \\
\underline{-4x - 1 \qquad\quad -4x \quad -1} & \text{Subtract } 4x \text{ and 1 from both sides} \\
0 = x^2 - 14x + 24 & \text{Factor} \\
0 = (x-12)(x-2) & \text{Set each factor equal to zero} \\
x - 12 = 0 \;\text{ or }\; x - 2 = 0 & \text{Solve each equation} \\
\underline{+12+12} \qquad \underline{+2+2} & \\
x = 12 \;\text{ or }\; x = 2 & \text{Need to check answers in original problem} \\
 & \\
(12) + \sqrt{4(12)+1} = 5 & \text{Check } x = 5 \text{ first}
\end{array}
$$

$$\begin{aligned}
12+\sqrt{48+1}&=5 && \text{Add}\\
12+\sqrt{49}&=5 && \text{Take root}\\
12+7&=5 && \text{Add}\\
19&=5 && \text{False, extraneous root}
\end{aligned}$$

$$\begin{aligned}
(2)+\sqrt{4(2)+1}&=5 && \text{Check } x=2\\
2+\sqrt{8+1}&=5 && \text{Add}\\
2+\sqrt{9}&=5 && \text{Take root}\\
2+3&=5 && \text{Add}\\
5&=5 && \text{True! It works}
\end{aligned}$$

$$x=2 \qquad \text{Our Solution}$$

The above example illustrates that as we solve we could end up with an x^2 term or a quadratic. In this case we remember to set the equation to zero and solve by factoring. We will have to check both solutions if the index in the problem was even. Sometimes both values work, sometimes only one, and sometimes neither works.

World View Note: The babylonians were the first known culture to solve quadratics in radicals - as early as 2000 BC!

If there is more than one square root in a problem we will clear the roots one at a time. This means we must first isolate one of them before we square both sides.

Example 446.

$$\begin{aligned}
\sqrt{3x-8}-\sqrt{x}&=0 && \text{Even index! We will have to check answers}\\
+\sqrt{x}\;+\sqrt{x}& && \text{Isolate first root by adding } \sqrt{x} \text{ to both sides}\\
\sqrt{3x-8}&=\sqrt{x} && \text{Square both sides}\\
(\sqrt{3x-8})^2&=(\sqrt{x})^2 && \text{Evaluate exponents}\\
3x-8&=x && \text{Solve}\\
-3x\quad\;&\;-3x && \text{Subtract } 3x \text{ from both sides}\\
-8&=-2x && \text{Divide both sides by } -2\\
\overline{-2}\quad&\;\;\overline{-2} && \\
4&=x && \text{Need to check answer in original}\\
\sqrt{3(4)-8}-\sqrt{4}&=0 && \text{Multiply}\\
\sqrt{12-8}-\sqrt{4}&=0 && \text{Subtract}\\
\sqrt{4}-\sqrt{4}&=0 && \text{Take roots}
\end{aligned}$$

$$
\begin{aligned}
2-2&=0 && \text{Subtract}\\
0&=0 && \text{True! It works}\\
x&=4 && \text{Our Solution}
\end{aligned}
$$

When there is more than one square root in the problem, after isolating one root and squaring both sides we may still have a root remaining in the problem. In this case we will again isolate the term with the second root and square both sides. When isolating, we will isolate the *term* with the square root. This means the square root can be multiplied by a number after isolating.

Example 447.

$$
\begin{aligned}
\sqrt{2x+1}-\sqrt{x}&=1 && \text{Even index! We will have to check answers}\\
\underline{\quad +\sqrt{x}+\sqrt{x}} & && \text{Isolate first root by adding } \sqrt{x} \text{ to both sides}\\
\sqrt{2x+1}&=\sqrt{x}+1 && \text{Square both sides}\\
(\sqrt{2x+1})^2&=(\sqrt{x}+1)^2 && \text{Evaluate exponents, recall } (a+b)^2=a^2+2ab+b^2\\
2x+1&=x+2\sqrt{x}+1 && \text{Isolate the term with the root}\\
\underline{-x-1-x \qquad\quad -1} & && \text{Subtract } x \text{ and } 1 \text{ from both sides}\\
x&=2\sqrt{x} && \text{Square both sides}\\
(x)^2&=(2\sqrt{x})^2 && \text{Evaluate exponents}\\
x^2&=4x && \text{Make equation equal zero}\\
-4x&-4x && \text{Subtract } x \text{ from both sides}\\
x^2-4x&=0 && \text{Factor}\\
x(x-4)&=0 && \text{Set each factor equal to zero}\\
x=0 \ \text{ or } \ x-4&=0 && \text{Solve}\\
\underline{+4+4} & && \text{Add 4 to both sides of second equation}\\
x=0 \ \text{ or } \ x&=4 && \text{Need to check answers in original}
\end{aligned}
$$

$$
\begin{aligned}
\sqrt{2(0)+1}-\sqrt{(0)}&=1 && \text{Check } x=0 \text{ first}\\
\sqrt{1}-\sqrt{0}&=1 && \text{Take roots}\\
1-0&=1 && \text{Subtract}\\
1&=1 && \text{True! It works}
\end{aligned}
$$

$$
\begin{aligned}
\sqrt{2(4)+1}-\sqrt{(4)}&=1 && \text{Check } x=4\\
\sqrt{8+1}-\sqrt{4}&=1 && \text{Add}\\
\sqrt{9}-\sqrt{4}&=1 && \text{Take roots}\\
3-2&=1 && \text{Subtract}\\
1&=1 && \text{True! It works}
\end{aligned}
$$

Sample PREVIEW

$$x = 0 \text{ or } 4 \qquad \text{Our Solution}$$

Example 448.

$$
\begin{array}{rl}
\sqrt{3x+9} - \sqrt{x+4} = -1 & \text{Even index! We will have to check answers} \\
\underline{+\sqrt{x+4} + \sqrt{x+4}} & \text{Isolate the first root by adding } \sqrt{x+4} \\
\sqrt{3x+9} = \sqrt{x+4} - 1 & \text{Square both sides} \\
(\sqrt{3x+9})^2 = (\sqrt{x+4} - 1)^2 & \text{Evaluate exponents} \\
3x + 9 = x + 4 - 2\sqrt{x+4} + 1 & \text{Combine like terms} \\
3x + 9 = x + 5 - 2\sqrt{x+4} & \text{Isolate the term with radical} \\
\underline{-x - 5 - x - 5} \qquad\qquad & \text{Subtract } x \text{ and } 5 \text{ from both sides} \\
2x + 4 = -2\sqrt{x+4} & \text{Square both sides} \\
(2x+4)^2 = (-2\sqrt{x+4})^2 & \text{Evaluate exponents} \\
4x^2 + 16x + 16 = 4(x+4) & \text{Distribute} \\
4x^2 + 16x + 16 = 4x + 16 & \text{Make equation equal zero} \\
\underline{-4x - 16 - 4x - 16} & \text{Subtract } 4x \text{ and } 16 \text{ from both sides} \\
4x^2 + 12x = 0 & \text{Factor} \\
4x(x+3) = 0 & \text{Set each factor equal to zero} \\
4x = 0 \text{ or } x + 3 = 0 & \text{Solve} \\
\overline{4} \quad \overline{4} \qquad \underline{-3 - 3} & \\
x = 0 \text{ or } x = -3 & \text{Check solutions in original}
\end{array}
$$

$$
\begin{array}{rl}
\sqrt{3(0)+9} - \sqrt{(0)+4} = -1 & \text{Check } x = 0 \text{ first} \\
\sqrt{9} - \sqrt{4} = -1 & \text{Take roots} \\
3 - 2 = -1 & \text{Subtract} \\
1 = -1 & \text{False, extraneous solution}
\end{array}
$$

$$
\begin{array}{rl}
\sqrt{3(-3)+9} - \sqrt{(-3)+4} = -1 & \text{Check } x = -3 \\
\sqrt{-9+9} - \sqrt{(-3)+4} = -1 & \text{Add} \\
\sqrt{0} - \sqrt{1} = -1 & \text{Take roots} \\
0 - 1 = -1 & \text{Subtract} \\
-1 = -1 & \text{True! It works}
\end{array}
$$

$$x = -3 \qquad \text{Our Solution}$$

9.1 Practice - Solving with Radicals

Solve.

1) $\sqrt{2x+3}-3=0$

2) $\sqrt{5x+1}-4=0$

3) $\sqrt{6x-5}-x=0$

4) $\sqrt{x+2}-\sqrt{x}=2$

5) $3+x=\sqrt{6x+13}$

6) $x-1=\sqrt{7-x}$

7) $\sqrt{3-3x}-1=2x$

8) $\sqrt{2x+2}=3+\sqrt{2x-1}$

9) $\sqrt{4x+5}-\sqrt{x+4}=2$

10) $\sqrt{3x+4}-\sqrt{x+2}=2$

11) $\sqrt{2x+4}-\sqrt{x+3}=1$

12) $\sqrt{7x+2}-\sqrt{3x+6}=6$

13) $\sqrt{2x+6}-\sqrt{x+4}=1$

14) $\sqrt{4x-3}-\sqrt{3x+1}=1$

15) $\sqrt{6-2x}-\sqrt{2x+3}=3$

16) $\sqrt{2-3x}-\sqrt{3x+7}=3$

Quadratics - Solving with Exponents

Objective: Solve equations with exponents using the odd root property and the even root property.

Another type of equation we can solve is one with exponents. As you might expect we can clear exponents by using roots. This is done with very few unexpected results when the exponent is odd. We solve these problems very straight forward using the odd root property

Odd Root Property: if $a^n = b$, then $a = \sqrt[n]{b}$ when n is odd

Example 449.

$$\begin{array}{rl} x^5 = 32 & \text{Use odd root property} \\ \sqrt[5]{x^5} = \sqrt[5]{32} & \text{Simplify roots} \\ x = 2 & \text{Our Solution} \end{array}$$

However, when the exponent is even we will have two results from taking an even root of both sides. One will be positive and one will be negative. This is because both $3^2 = 9$ and $(-3)^2 = 9$. so when solving $x^2 = 9$ we will have two solutions, one positive and one negative: $x = 3$ and -3

Even Root Property: if $a^n = b$, then $a = \pm \sqrt[n]{b}$ when n is even

Example 450.

$$\begin{array}{rl} x^4 = 16 & \text{Use even root property } (\pm) \end{array}$$

$$
\begin{array}{rl}
\sqrt[4]{x^4} = \pm\sqrt[4]{16} & \text{Simplify roots} \\
x = \pm 2 & \text{Our Solution}
\end{array}
$$

World View Note: In 1545, French Mathematicain Gerolamo Cardano published his book *The Great Art, or the Rules of Algebra* which included the solution of an equation with a fourth power, but it was considered absurd by many to take a quantity to the fourth power because there are only three dimensions!

Example 451.

$$
\begin{array}{rl}
(2x+4)^2 = 36 & \text{Use even root property } (\pm) \\
\sqrt{(2x+4)^2} = \pm\sqrt{36} & \text{Simplify roots} \\
2x+4 = \pm 6 & \text{To avoid sign errors we need two equations} \\
2x+4 = 6 \ \text{ or } \ 2x+4 = -6 & \text{One equation for } +\text{, one equation for } - \\
\underline{-4 \ -4} \qquad \underline{-4 \quad -4} & \text{Subtract 4 from both sides} \\
2x = 2 \ \text{ or } \ 2x = -10 & \text{Divide both sides by 2} \\
\overline{2} \quad \overline{2} \qquad \overline{2} \quad \overline{2} & \\
x = 1 \ \text{ or } \ x = -5 & \text{Our Solutions}
\end{array}
$$

In the previous example we needed two equations to simplify because when we took the root, our solutions were two rational numbers, 6 and -6. If the roots did not simplify to rational numbers we can keep the $\pm$ in the equation.

Example 452.

$$
\begin{array}{rl}
(6x-9)^2 = 45 & \text{Use even root property } (\pm) \\
\sqrt{(6x-9)^2} = \pm\sqrt{45} & \text{Simplify roots} \\
6x - 9 = \pm 3\sqrt{5} & \text{Use one equation because root did not simplify to rational} \\
\underline{+9 \ +9} & \text{Add 9 to both sides} \\
6x = 9 \pm 3\sqrt{5} & \text{Divide both sides by 6} \\
\overline{6} \qquad \overline{\quad 6 \quad} & \\
x = \dfrac{9 \pm 3\sqrt{5}}{6} & \text{Simplify, divide each term by 3} \\
x = \dfrac{3 \pm \sqrt{5}}{2} & \text{Our Solution}
\end{array}
$$

When solving with exponents, it is important to first isolate the part with the exponent before taking any roots.

Example 453.

$$\begin{array}{rl}
(x+4)^3-6=119 & \text{Isolate part with exponent}\\
\underline{+6\quad +6} & \\
(x+4)^3=125 & \text{Use odd root property}\\
\sqrt[3]{(x+4)^3}=\sqrt{125} & \text{Simplify roots}\\
x+4=5 & \text{Solve}\\
\underline{-4-4} & \text{Subtract 4 from both sides}\\
x=1 & \text{Our Solution}
\end{array}$$

Example 454.

$$\begin{array}{rl}
(6x+1)^2+6=10 & \text{Isolate part with exponent}\\
\underline{-6-6} & \text{Subtract 6 from both sides}\\
(6x+1)^2=4 & \text{Use even root property } (\pm)\\
\sqrt{(6x+1)^2}=\pm\sqrt{4} & \text{Simplify roots}\\
6x+1=\pm 2 & \text{To avoid sign errors, we need two equations}\\
6x+1=2 \quad\text{or}\quad 6x+1=-2 & \text{Solve each equation}\\
\underline{-1-1} \qquad\quad \underline{-1\quad -1} & \text{Subtract 1 from both sides}\\
6x=1 \quad\text{or}\quad 6x=-3 & \text{Divide both sides by 6}\\
\overline{6}\quad \overline{6} \qquad \overline{6}\quad \overline{6} & \\
x=\frac{1}{6} \quad\text{or}\quad x=-\frac{1}{2} & \text{Our Solution}
\end{array}$$

When our exponents are a fraction we will need to first convert the fractional exponent into a radical expression to solve. Recall that $a^{\frac{m}{n}}=(\sqrt[n]{a})^m$. Once we have done this we can clear the exponent using either the even ($\pm$) or odd root property. Then we can clear the radical by raising both sides to an exponent (remember to check answers if the index is even).

Example 455.

$$\begin{array}{rl}
(4x+1)^{\frac{2}{5}}=9 & \text{Rewrite as a radical expression}\\
(\sqrt[5]{4x+1})^2=9 & \text{Clear exponent first with even root property } (\pm)\\
\sqrt{(\sqrt[5]{4x+1})^2}=\pm\sqrt{9} & \text{Simplify roots}
\end{array}$$

$$
\begin{array}{rl}
\sqrt[5]{4x+1} = \pm 3 & \text{Clear radical by raising both sides to 5th power} \\
(\sqrt[5]{4x+1})^5 = (\pm 3)^5 & \text{Simplify exponents} \\
4x+1 = \pm 243 & \text{Solve, need 2 equations!} \\
4x+1 = 243 \text{ or } 4x+1 = -243 & \\
\underline{-1 \quad -1} \qquad \underline{-1 \quad -1} & \text{Subtract 1 from both sides} \\
\frac{4x}{4} = \frac{242}{4} \text{ or } \frac{4x}{4} = \frac{-244}{4} & \text{Divide both sides by 4} \\
x = \frac{121}{2}, -61 & \text{Our Solution}
\end{array}
$$

Example 456.

$$
\begin{array}{rl}
(3x-2)^{\frac{3}{4}} = 64 & \text{Rewrite as radical expression} \\
(\sqrt[4]{3x-2})^3 = 64 & \text{Clear exponent first with odd root property} \\
\sqrt[3]{(\sqrt[4]{3x-2})^3} = \sqrt[3]{64} & \text{Simplify roots} \\
\sqrt[4]{3x-2} = 4 & \text{Even Index! Check answers.} \\
(\sqrt[4]{3x-2})^4 = 4^4 & \text{Raise both sides to 4th power} \\
3x-2 = 256 & \text{Solve} \\
\underline{+2 \quad +2} & \text{Add 2 to both sides} \\
\frac{3x}{3} = \frac{258}{3} & \text{Divide both sides by 3} \\
x = 86 & \text{Need to check answer in radical form of problem} \\
(\sqrt[4]{3(86)-2})^3 = 64 & \text{Multiply} \\
(\sqrt[4]{258-2})^3 = 64 & \text{Subtract} \\
(\sqrt[4]{256})^3 = 64 & \text{Evaluate root} \\
4^3 = 64 & \text{Evaluate exponent} \\
64 = 64 & \text{True! It works} \\
x = 86 & \text{Our Solution}
\end{array}
$$

With rational exponents it is very helpful to convert to radical form to be able to see if we need a $\pm$ because we used the even root property, or to see if we need to check our answer because there was an even root in the problem. When checking we will usually want to check in the radical form as it will be easier to evaluate.

9.2 Practice - Solving with Exponents

Solve.

1) $x^2 = 75$

2) $x^3 = -8$

3) $x^2 + 5 = 13$

4) $4x^3 - 2 = 106$

5) $3x^2 + 1 = 73$

6) $(x-4)^2 = 49$

7) $(x+2)^5 = -243$

8) $(5x+1)^4 = 16$

9) $(2x+5)^3 - 6 = 21$

10) $(2x+1)^2 + 3 = 21$

11) $(x-1)^{\frac{2}{3}} = 16$

12) $(x-1)^{\frac{3}{2}} = 8$

13) $(2-x)^{\frac{3}{2}} = 27$

14) $(2x+3)^{\frac{4}{3}} = 16$

15) $(2x-3)^{\frac{2}{3}} = 4$

16) $(x+3)^{-\frac{1}{3}} = 4$

17) $\left(x+\frac{1}{2}\right)^{-\frac{2}{3}} = 4$

18) $(x-1)^{-\frac{5}{3}} = 32$

19) $(x-1)^{-\frac{5}{2}} = 32$

20) $(x+3)^{\frac{3}{2}} = -8$

21) $(3x-2)^{\frac{4}{5}} = 16$

22) $(2x+3)^{\frac{3}{2}} = 27$

23) $(4x+2)^{\frac{3}{5}} = -8$

24) $(3-2x)^{\frac{4}{3}} = -81$

9.3

Quadratics - Complete the Square

Objective: Solve quadratic equations by completing the square.

When solving quadratic equations in the past we have used factoring to solve for our variable. This is exactly what is done in the next example.

Example 457.

$$\begin{array}{rl} x^2+5x+6=0 & \text{Factor} \\ (x+3)(x+2)=0 & \text{Set each factor equal to zero} \\ x+3=0 \quad \text{or} \quad x+2=0 & \text{Solve each equation} \\ \underline{-3-3} \qquad\qquad \underline{-2-2} & \\ x=-3 \quad \text{or} \quad x=-2 & \text{Our Solutions} \end{array}$$

However, the problem with factoring is all equations cannot be factored. Consider the following equation: $x^2-2x-7=0$. The equation cannot be factored, however there are two solutions to this equation, $1+2\sqrt{2}$ and $1-2\sqrt{2}$. To find these two solutions we will use a method known as completing the square. When completing the square we will change the quadratic into a perfect square which can easily be solved with the square root property. The next example reviews the square root property.

Example 458.

$$\begin{array}{rl} (x+5)^2=18 & \text{Square root of both sides} \\ \sqrt{(x+5)^2}=\pm\sqrt{18} & \text{Simplify each radical} \\ x+5=\pm 3\sqrt{2} & \text{Subtract 5 from both sides} \\ \underline{-5 \quad -5} & \\ x=-5\pm 3\sqrt{2} & \text{Our Solution} \end{array}$$

To complete the square, or make our problem into the form of the previous example, we will be searching for the third term in a trinomial. If a quadratic is of the form $x^2 + bx + c$, and a perfect square, the third term, c, can be easily found by the formula $\left(\frac{1}{2} \cdot b\right)^2$. This is shown in the following examples, where we find the number that completes the square and then factor the perfect square.

Example 459.

$$x^2 + 8x + c \qquad c = \left(\frac{1}{2} \cdot b\right)^2 \text{ and our } b = 8$$

$$\left(\frac{1}{2} \cdot 8\right)^2 = 4^2 = 16 \qquad \text{The third term to complete the square is } 16$$

$$x^2 + 8x + 16 \qquad \text{Our equation as } a \text{ perfect square, factor}$$

$$(x+4)^2 \qquad \text{Our Solution}$$

Example 460.

$$x^2 - 7x + c \qquad c = \left(\frac{1}{2} \cdot b\right)^2 \text{ and our } b = 7$$

$$\left(\frac{1}{2} \cdot 7\right)^2 = \left(\frac{7}{2}\right)^2 = \frac{49}{4} \qquad \text{The third term to complete the square is } \frac{49}{4}$$

$$x^2 - 11x + \frac{49}{4} \qquad \text{Our equation as } a \text{ perfect square, factor}$$

$$\left(x - \frac{7}{2}\right)^2 \qquad \text{Our Solution}$$

Example 461.

$$x^2 + \frac{5}{3}x + c \qquad c = \left(\frac{1}{2} \cdot b\right)^2 \text{ and our } b = 8$$

$$\left(\frac{1}{2} \cdot \frac{5}{3}\right)^2 = \left(\frac{5}{6}\right)^2 = \frac{25}{36} \qquad \text{The third term to complete the square is } \frac{25}{36}$$

$$x^2 + \frac{5}{3}x + \frac{25}{36} \quad \text{Our equation as a perfect square, factor}$$

$$\left(x + \frac{5}{6}\right)^2 \quad \text{Our Solution}$$

The process in the previous examples, combined with the even root property, is used to solve quadratic equations by completing the square. The following five steps describe the process used to complete the square, along with an example to demonstrate each step.

Problem	$3x^2 + 18x - 6 = 0$
1. Separate constant term from variables	$\underline{+6+6}$ $3x^2 + 18x \quad = 6$
2. Divide each term by a	$\frac{3}{3}x^2 + \frac{18}{3}x \quad = \frac{6}{3}$ $x^2 + 6x \quad = 2$
3. Find value to complete the square: $\left(\frac{1}{2} \cdot b\right)^2$	$\left(\frac{1}{2} \cdot 6\right)^2 = 3^2 = 9$
4. Add to both sides of equation	$x^2 + 6x \quad = 2$ $\underline{+9 \ +9}$ $x^2 + 6x + 9 = 11$
5. Factor	$(x+3)^2 = 11$
Solve by even root property	$\sqrt{(x+3)^2} = \pm\sqrt{11}$ $x + 3 = \pm\sqrt{11}$ $\underline{-3 \ -3}$ $x = -3 \pm \sqrt{11}$

World View Note: The Chinese in 200 BC were the first known culture group to use a method similar to completing the square, but their method was only used to calculate positive roots.

The advantage of this method is it can be used to solve any quadratic equation. The following examples show how completing the square can give us rational solutions, irrational solutions, and even complex solutions.

Example 462.

$$2x^2 + 20x + 48 = 0 \quad \text{Separate constant term from varaibles}$$

$$\begin{array}{rl}
\underline{-48-48} & \text{Subtract 24} \\
2x^2+20x \quad = -48 & \text{Divide by } a \text{ or } 2 \\
\frac{2x^2}{2}+\frac{20x}{2} = \frac{-48}{2} & \\
x^2+10x \quad = -24 & \text{Find number to complete the square: } \left(\frac{1}{2}\cdot b\right)^2 \\
\left(\frac{1}{2}\cdot 10\right)^2 = 5^2 = 25 & \text{Add 25 to both sides of the equation} \\
x^2+10x \quad = -24 & \\
\underline{+25 \quad +25} & \\
x^2+10x+25 = 1 & \text{Factor} \\
(x+5)^2 = 1 & \text{Solve with even root property} \\
\sqrt{(x+5)^2} = \pm\sqrt{1} & \text{Simplify roots} \\
x+5 = \pm 1 & \text{Subtract 5 from both sides} \\
\underline{-5-5} & \\
x = -5 \pm 1 & \text{Evaluate} \\
x = -4 \text{ or } -6 & \text{Our Solution}
\end{array}$$

Example 463.

$$\begin{array}{rl}
x^2-3x-2=0 & \text{Separate constant from variables} \\
\underline{+2+2} & \text{Add 2 to both sides} \\
x^2-3x \quad = 2 & \text{No } a\text{, find number to complete the square } \left(\frac{1}{2}\cdot b\right)^2 \\
\left(\frac{1}{2}\cdot 3\right)^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4} & \text{Add } \frac{9}{4} \text{ to both sides,} \\
\frac{2}{1}\left(\frac{4}{4}\right)+\frac{9}{4} = \frac{8}{4}+\frac{9}{4} = \frac{17}{4} & \text{Need common denominator (4) on right} \\
x^2-3x+\frac{9}{4} = \frac{8}{4}+\frac{9}{4} = \frac{17}{4} & \text{Factor} \\
\left(x-\frac{3}{2}\right)^2 = \frac{17}{4} & \text{Solve using the even root property} \\
\sqrt{\left(x-\frac{3}{2}\right)^2} = \pm\sqrt{\frac{17}{4}} & \text{Simplify roots} \\
x-\frac{3}{2} = \frac{\pm\sqrt{17}}{2} & \text{Add } \frac{3}{2} \text{ to both sides,}
\end{array}$$

$$\underline{+\frac{3}{2}+\frac{3}{2}}$$ we already have a common denominator

$$x=\frac{3\pm\sqrt{17}}{2}$$ Our Solution

Example 464.

$$3x^2=2x-7$$ Separate the constant from the variables

$$\underline{-2x-2x}$$ Subtract $2x$ from both sides

$$\frac{3x^2}{3}-\frac{2x}{3}=\frac{-7}{3}$$ Divide each term by a or 3

$$x^2-\frac{2}{3}x\quad=-\frac{7}{3}$$ Find the number to complete the square $\left(\frac{1}{2}\cdot b\right)^2$

$$\left(\frac{1}{2}\cdot\frac{2}{3}\right)^2=\left(\frac{1}{3}\right)^2=\frac{1}{9}$$ Add to both sides,

$$-\frac{7}{3}\left(\frac{3}{3}\right)+\frac{1}{9}=\frac{-21}{3}+\frac{1}{9}=\frac{-20}{9}$$ get common denominator on right

$$x^2-\frac{2}{3}x+\frac{1}{3}=-\frac{20}{9}$$ Factor

$$\left(x-\frac{1}{3}\right)^2=-\frac{20}{9}$$ Solve using the even root property

$$\sqrt{\left(x-\frac{1}{3}\right)^2}=\pm\sqrt{\frac{-20}{9}}$$ Simplify roots

$$x-\frac{1}{3}=\frac{\pm 2i\sqrt{5}}{3}$$ Add $\frac{1}{3}$ to both sides,

$$\underline{+\frac{1}{3}+\frac{1}{3}}$$ Already have common denominator

$$x=\frac{1\pm 2i\sqrt{5}}{3}$$ Our Solution

As several of the examples have shown, when solving by completing the square we will often need to use fractions and be comfortable finding common denominators and adding fractions together. Once we get comfortable solving by completing the square and using the five steps, any quadratic equation can be easily solved.

9.3 Practice - Complete the Square

Find the value that completes the square and then rewrite as a perfect square.

1) $x^2 - 30x + \_\_$

2) $a^2 - 24a + \_\_$

3) $m^2 - 36m + \_\_$

4) $x^2 - 34x + \_\_$

5) $x^2 - 15x + \_\_$

6) $r^2 - \frac{1}{9}r + \_\_$

7) $y^2 - y + \_\_$

8) $p^2 - 17p + \_\_$

Solve each equation by completing the square.

9) $x^2 - 16x + 55 = 0$

10) $n^2 - 8n - 12 = 0$

11) $v^2 - 8v + 45 = 0$

12) $b^2 + 2b + 43 = 0$

13) $6x^2 + 12x + 63 = 0$

14) $3x^2 - 6x + 47 = 0$

15) $5k^2 - 10k + 48 = 0$

16) $8a^2 + 16a - 1 = 0$

17) $x^2 + 10x - 57 = 4$

18) $p^2 - 16p - 52 = 0$

19) $n^2 - 16n + 67 = 4$

20) $m^2 - 8m - 3 = 6$

21) $2x^2 + 4x + 38 = -6$

22) $6r^2 + 12r - 24 = -6$

23) $8b^2 + 16b - 37 = 5$

24) $6n^2 - 12n - 14 = 4$

25) $x^2 = -10x - 29$

26) $v^2 = 14v + 36$

27) $n^2 = -21 + 10n$

28) $a^2 - 56 = -10a$

29) $3k^2 + 9 = 6k$

30) $5x^2 = -26 + 10x$

31) $2x^2 + 63 = 8x$

32) $5n^2 = -10n + 15$

33) $p^2 - 8p = -55$

34) $x^2 + 8x + 15 = 8$

35) $7n^2 - n + 7 = 7n + 6n^2$

36) $n^2 + 4n = 12$

37) $13b^2 + 15b + 44 = -5 + 7b^2 + 3b$

38) $-3r^2 + 12r + 49 = -6r^2$

39) $5x^2 + 5x = -31 - 5x$

40) $8n^2 + 16n = 64$

41) $v^2 + 5v + 28 = 0$

42) $b^2 + 7b - 33 = 0$

43) $7x^2 - 6x + 40 = 0$

44) $4x^2 + 4x + 25 = 0$

45) $k^2 - 7k + 50 = 3$

46) $a^2 - 5a + 25 = 3$

47) $5x^2 + 8x - 40 = 8$

48) $2p^2 - p + 56 = -8$

49) $m^2 = -15 + 9m$

50) $n^2 - n = -41$

51) $8r^2 + 10r = -55$

52) $3x^2 - 11x = -18$

53) $5n^2 - 8n + 60 = -3n + 6 + 4n^2$

54) $4b^2 - 15b + 56 = 3b^2$

55) $-2x^2 + 3x - 5 = -4x^2$

56) $10v^2 - 15v = 27 + 4v^2 - 6v$

Quadratics - Quadratic Formula

Objective: Solve quadratic equations by using the quadratic formula.

The general from of a quadratic is $ax^2 + bx + c = 0$. We will now solve this formula for x by completing the square

Example 465.

$$
\begin{array}{rl}
ax^2 + bc + c = 0 & \text{Separate constant from variables} \\
\underline{-c - c} & \text{Subtract } c \text{ from both sides} \\
\dfrac{ax^2}{a} + \dfrac{bx}{a} \quad = \dfrac{-c}{a} & \text{Divide each term by } a \\
x^2 + \dfrac{b}{a}x \quad = \dfrac{-c}{a} & \text{Find the number that completes the square} \\
\left(\dfrac{1}{2}\cdot\dfrac{b}{a}\right)^2 = \left(\dfrac{b}{2a}\right)^2 = \dfrac{b^2}{4a^2} & \text{Add to both sides,} \\
\dfrac{b^2}{4a^2} - \dfrac{c}{a}\left(\dfrac{4a}{4a}\right) = \dfrac{b^2}{4a^2} - \dfrac{4ac}{4a^2} = \dfrac{b^2 - 4ac}{4a^2} & \text{Get common denominator on right} \\
x^2 + \dfrac{b}{a}x + \dfrac{b^2}{4a^2} = \dfrac{b^2}{4a^2} - \dfrac{4ac}{4a^2} = \dfrac{b^2 - 4ac}{4a^2} & \text{Factor} \\
\left(x + \dfrac{b}{2a}\right)^2 = \dfrac{b^2 - 4ac}{4a^2} & \text{Solve using the even root property} \\
\sqrt{\left(x + \dfrac{b}{2a}\right)^2} = \pm\sqrt{\dfrac{b^2 - 4ac}{4a^2}} & \text{Simplify roots} \\
x + \dfrac{b}{2a} = \dfrac{\pm\sqrt{b^2 - 4ac}}{2a} & \text{Subtract } \dfrac{b}{2a} \text{ from both sides} \\
x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} & \text{Our Solution}
\end{array}
$$

This solution is a very important one to us. As we solved a general equation by completing the square, we can use this formula to solve any quadratic equation. Once we identify what a, b, and c are in the quadratic, we can substitute those values into $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and we will get our two solutions. This formula is known as the quadratic fromula

$$\textbf{Quadratic Formula: if } ax^2 + bx + c = 0 \textbf{ then } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

World View Note: Indian mathematician Brahmagupta gave the first explicit formula for solving quadratics in 628. However, at that time mathematics was not done with variables and symbols, so the formula he gave was, "To the absolute number multiplied by four times the square, add the square of the middle term; the square root of the same, less the middle term, being divided by twice the square is the value." This would translate to $\frac{\sqrt{4ac + b^2} - b}{2a}$ as the solution to the equation $ax^2 + bx = c$.

We can use the quadratic formula to solve any quadratic, this is shown in the following examples.

Example 466.

$$\begin{aligned}
x^2 + 3x + 2 &= 0 && a = 1, b = 3, c = 2, \text{use quadratic formula}\\
x &= \frac{-3 \pm \sqrt{3^2 - 4(1)(2)}}{2(1)} && \text{Evaluate exponent and multiplication}\\
x &= \frac{-3 \pm \sqrt{9 - 8}}{2} && \text{Evaluate subtraction under root}\\
x &= \frac{-3 \pm \sqrt{1}}{2} && \text{Evaluate root}\\
x &= \frac{-3 \pm 1}{2} && \text{Evaluate} \pm \text{to get two answers}\\
x &= \frac{-2}{2} \text{ or } \frac{-4}{2} && \text{Simplify fractions}\\
x &= -1 \text{ or } -2 && \text{Our Solution}
\end{aligned}$$

As we are solving using the quadratic formula, it is important to remember the equation must fist be equal to zero.

Example 467.

$$\begin{aligned}
25x^2 &= 30x + 11 && \text{First set equal to zero}\\
-30x - 11 &\quad -30x - 11 && \text{Subtract } 30x \text{ and } 11 \text{ from both sides}\\
25x^2 - 30x - 11 &= 0 && a = 25, b = -30, c = -11, \text{use quadratic formula}\\
x &= \frac{30 \pm \sqrt{(-30)^2 - 4(25)(-11)}}{2(25)} && \text{Evaluate exponent and multiplication}
\end{aligned}$$

$$
\begin{aligned}
x &= \frac{30 \pm \sqrt{900 + 1100}}{50} && \text{Evaluate addition inside root}\\
x &= \frac{30 \pm \sqrt{2000}}{50} && \text{Simplify root}\\
x &= \frac{30 \pm 20\sqrt{5}}{50} && \text{Reduce fraction by dividing each term by 10}\\
x &= \frac{3 \pm 2\sqrt{5}}{5} && \text{Our Solution}
\end{aligned}
$$

Example 468.

$$
\begin{aligned}
3x^2 + 4x + 8 &= 2x^2 + 6x - 5 && \text{First set equation equal to zero}\\
-2x^2 - 6x + 5 &\quad -2x^2 - 6x + 5 && \text{Subtract } 2x^2 \text{ and } 6x \text{ and add } 5\\
x^2 - 2x + 13 &= 0 && a = 1, b = -2, c = 13, \text{ use quadratic formula}\\
x &= \frac{2 \pm \sqrt{(-2)^2 - 4(1)(13)}}{2(1)} && \text{Evaluate exponent and multiplication}\\
x &= \frac{2 \pm \sqrt{4 - 52}}{2} && \text{Evaluate subtraction inside root}\\
x &= \frac{2 \pm \sqrt{-48}}{2} && \text{Simplify root}\\
x &= \frac{2 \pm 4i\sqrt{3}}{2} && \text{Reduce fraction by dividing each term by 2}\\
x &= 1 \pm 2i\sqrt{3} && \text{Our Solution}
\end{aligned}
$$

When we use the quadratic formula we don't necessarily get two unique answers. We can end up with only one solution if the square root simplifies to zero.

Example 469.

$$
\begin{aligned}
4x^2 - 12x + 9 &= 0 && a = 4, b = -12, c = 9, \text{ use quadratic formula}\\
x &= \frac{12 \pm \sqrt{(-12)^2 - 4(4)(9)}}{2(4)} && \text{Evaluate exponents and multiplication}\\
x &= \frac{12 \pm \sqrt{144 - 144}}{8} && \text{Evaluate subtraction inside root}\\
x &= \frac{12 \pm \sqrt{0}}{8} && \text{Evaluate root}\\
x &= \frac{12 \pm 0}{8} && \text{Evaluate } \pm\\
x &= \frac{12}{8} && \text{Reduce fraction}\\
x &= \frac{3}{2} && \text{Our Solution}
\end{aligned}
$$

If a term is missing from the quadratic, we can still solve with the quadratic formula, we simply use zero for that term. The order is important, so if the term with x is missing, we have $b=0$, if the constant term is missing, we have $c=0$.

Example 470.

$3x^2+7=0$	$a=3, b=0$ (missing term), $c=7$
$x=\dfrac{-0\pm\sqrt{0^2-4(3)(7)}}{2(3)}$	Evaluate exponnets and multiplication, zeros not needed
$x=\dfrac{\pm\sqrt{-84}}{6}$	Simplify root
$x=\dfrac{\pm 2i\sqrt{21}}{6}$	Reduce, dividing by 2
$x=\dfrac{\pm i\sqrt{21}}{3}$	Our Solution

We have covered three different methods to use to solve a quadratic: factoring, complete the square, and the quadratic formula. It is important to be familiar with all three as each has its advantage to solving quadratics. The following table walks through a suggested process to decide which method would be best to use for solving a problem.

1. If it can easily factor, solve by factoring	$x^2-5x+6=0$ $(x-2)(x-3)=0$ $x=2$ or $x=3$
2. If $a=1$ and b is even, complete the square	$x^2+2x=4$ $\left(\frac{1}{2}\cdot 2\right)^2=1^2=1$ $x^2+2x+1=5$ $(x+1)^2=5$ $x+1=\pm\sqrt{5}$ $x=-1\pm\sqrt{5}$
3. Otherwise, solve by the quadratic formula	$x^2-3x+4=0$ $x=\frac{3\pm\sqrt{(-3)^2-4(1)(4)}}{2(1)}$ $x=\frac{3\pm i\sqrt{7}}{2}$

The above table is mearly a suggestion for deciding how to solve a quadtratic. Remember completing the square and quadratic formula will always work to solve any quadratic. Factoring only woks if the equation can be factored.

9.4 Practice - Quadratic Formula

Solve each equation with the quadratic formula.

1) $4a^2+6=0$

2) $3k^2+2=0$

3) $2x^2-8x-2=0$

4) $6n^2-1=0$

5) $2m^2-3=0$

6) $5p^2+2p+6=0$

7) $3r^2-2r-1=0$

8) $2x^2-2x-15=0$

9) $4n^2-36=0$

10) $3b^2+6=0$

11) $v^2-4v-5=-8$

12) $2x^2+4x+12=8$

13) $2a^2+3a+14=6$

14) $6n^2-3n+3=-4$

15) $3k^2+3k-4=7$

16) $4x^2-14=-2$

17) $7x^2+3x-16=-2$

18) $4n^2+5n=7$

19) $2p^2+6p-16=4$

20) $m^2+4m-48=-3$

21) $3n^2+3n=-3$

22) $3b^2-3=8b$

23) $2x^2=-7x+49$

24) $3r^2+4=-6r$

25) $5x^2=7x+7$

26) $6a^2=-5a+13$

27) $8n^2=-3n-8$

28) $6v^2=4+6v$

29) $2x^2+5x=-3$

30) $x^2=8$

31) $4a^2-64=0$

32) $2k^2+6k-16=2k$

33) $4p^2+5p-36=3p^2$

34) $12x^2+x+7=5x^2+5x$

35) $-5n^2-3n-52=2-7n^2$

36) $7m^2-6m+6=-m$

37) $7r^2-12=-3r$

38) $3x^2-3=x^2$

39) $2n^2-9=4$

40) $6b^2=b^2+7-b$

Sample PREVIEW

42) 1.2×10^6

5.4

Answers to Introduction to Polynomials

1) 3

2) 7

3) -10

4) -6

5) -7

6) 8

7) 5

8) -1

9) 12

10) -1

11) $3p^4-3p$

12) $-m^3+12m^2$

13) $-n^3+10n^2$

14) $8x^3+8x^2$

15) $5n^4+5n$

16) $2v^4+6$

17) $13p^3$

18) $-3x$

19) $3n^3+8$

20) x^4+9x^2-5

21) $2b^4+2b+10$

22) $-3r^4+12r^2-1$

23) $-5x^4+14x^3-1$

24) $5n^4-4n+7$

25) $7a^4-3a^2-2a$

26) $12v^3+3v+3$

27) p^2+4p-6

28) $3m^4-2m+6$

29) $5b^3+12b^2+5$

30) $-15n^4+4n-6$

31) n^3-5n^2+3

32) $-6x^4+13x^3$

33) $-12n^4+n^2+7$

34) $9x^2+10x^2$

35) $r^4-3r^3+7r^2+1$

36) $10x^3-6x^2+3x-8$

37) $9n^4+2n^3+6n^2$

38) $2b^4-b^3+4b^2+4b$

39) $-3b^4+13b^3-7b^2-11b+19$

40) $12n^4-n^3-6n^2+10$

41) $2x^4-x^3-4x+2$

42) $3x^4+9x^2+4x$

5.5

Answers to Multiply Polynomials

1) $6p-42$

2) $32k^2+16k$

3) $12x+6$

4) $18n^3+21n^2$

5) $20m^5+20m^4$

6) $12r-21$

7) $32n^2+80n+48$

8) $2x^2-7x-4$

9) $56b^2-19b-15$

10) $4r^2+40r+64$

11) $8x^2+22x+15$

12) $7n^2+43n-42$

13) $15v^2-26v+8$

14) $6a^2-44a-32$

15) $24x^2-22x-7$

16) $20x^2-29x+6$

17) $30x^2-14xy-4y^2$

18) $16u^2+10uv-21v^2$

19) $3x^2+13xy+12y^2$

20) $40u^2-34uv-48v^2$

21) $56x^2+61xy+15y^2$

22) $5a^2-7ab-24b^2$

23) $6r^3-43r^2+12r-35$

8.8

Answers - Complex Numbers

1) $11-4i$
2) $-4i$
3) $-3+9i$
4) $-1-6i$
5) $-3-13i$
6) $5-12i$
7) $-4-11i$
8) $-3-6i$
9) $-8-2i$
10) $13-8i$
11) 48
12) 24
13) 40
14) 32
15) -49
16) $28-21i$
17) $11+60i$
18) $-32-128i$
19) $80-10i$
20) $36-36i$
21) $27+38i$
22) $-28+76i$
23) $44+8i$
24) $16-18i$
25) $-3+11i$
26) $-1+13i$
27) $9i+5$
28) $\frac{-3i-2}{3}$
29) $\frac{10i-9}{6}$
30) $\frac{4i+2}{3}$
31) $\frac{3i-6}{4}$
32) $\frac{5i+9}{9}$
33) $10i+1$
34) $-2i$
35) $\frac{-40i+4}{101}$
36) $\frac{9i-45}{26}$
37) $\frac{56+48i}{85}$
38) $\frac{4-6i}{13}$
39) $\frac{70+49i}{149}$
40) $\frac{-36+27i}{50}$
41) $\frac{-30i-5}{37}$
42) $\frac{48i-56}{85}$
43) $9i$
44) $3i\sqrt{5}$
45) $-2\sqrt{5}$
46) $-2\sqrt{6}$
47) $\frac{1+i\sqrt{3}}{2}$
48) $\frac{2+i\sqrt{2}}{2}$
49) $2-i$
50) $\frac{3+2i\sqrt{2}}{2}$
51) i
52) $-i$
53) 1
54) 1
55) -1
56) i
57) -1
58) $-i$

Answers - Chapter 9

9.1

Answers - Solving with Radicals

1) 3
2) 3
3) 1, 5
4) no solution
5) ± 2
6) 3
7) $\frac{1}{4}$
8) no solution
9) 5
10) 7
11) 6
12) 46
13) 5
14) 21

15) $-\frac{3}{2}$

16) $-\frac{7}{3}$

9.2

Answers - Solving with Exponents

1) $\pm 5\sqrt{3}$

2) -2

3) $\pm 2\sqrt{2}$

4) 3

5) $\pm 2\sqrt{6}$

6) $-3, 11$

7) -5

8) $\frac{1}{5}, -\frac{3}{5}$

9) -1

10) $\frac{-1 \pm 3\sqrt{2}}{2}$

11) $65, -63$

12) 5

13) -7

14) $-\frac{11}{2}, \frac{5}{2}$

15) $\frac{11}{2}, -\frac{5}{2}$

16) $-\frac{191}{64}$

17) $-\frac{3}{8}, -\frac{5}{8}$

18) $\frac{9}{8}$

19) $\frac{5}{4}$

20) No Solution

21) $-\frac{34}{3}, -10$

22) 3

23) $-\frac{17}{2}$

24) No Solutoin

9.3

Answers - Complete the Square

1) 225; $(x-15)^2$

2) 144; $(a-12)^2$

3) 324; $(m-18)^2$

4) 289; $(x-17)^2$

5) $\frac{225}{4}$; $(x-\frac{15}{2})^2$

6) $\frac{1}{324}$; $(r-\frac{1}{18})^2$

7) $\frac{1}{4}$; $(y-\frac{1}{2})^2$

8) $\frac{289}{4}$; $(p-\frac{17}{2})^2$

9) $11, 5$

10) $4+2\sqrt{7}, 4-2\sqrt{7}$

11) $4+i\sqrt{29}, 4-i\sqrt{29}$

12) $-1+i\sqrt{42}, -1-i\sqrt{42}$

13) $\frac{-2+i\sqrt{38}}{2}, \frac{-2-i\sqrt{38}}{2}$

14) $\frac{3+2i\sqrt{33}}{3}, \frac{3-2i\sqrt{33}}{3}$

15) $\frac{5+i\sqrt{215}}{5}, \frac{5-i\sqrt{215}}{5}$

16) $\frac{-4+3\sqrt{2}}{4}, \frac{-4-3\sqrt{2}}{4}$

17) $-5+\sqrt{86}, -5-\sqrt{86}$

18) $8+2\sqrt{29}, 8-2\sqrt{29}$

19) $9, 7$

20) $9, -1$

21) $-1+i\sqrt{21}, -1-i\sqrt{21}$

22) $1, -3$

23) $\frac{3}{2}, -\frac{7}{2}$

24) $3, -1$

25) $-5+2i, -5-2i$

26) $7+\sqrt{85}, 7-\sqrt{85}$

27) $7, 3$

28) $4, -14$

29) $1+i\sqrt{2}, 1-i\sqrt{2}$

30) $\frac{5+i\sqrt{105}}{5}, \frac{5-i\sqrt{105}}{5}$

31) $\frac{4+i\sqrt{110}}{2}, \frac{4-i\sqrt{110}}{2}$

32) $1, -3$

33) $4+i\sqrt{39}, 4-i\sqrt{39}$

34) $-1.-7$

35) $7, 1$

36) $2, -6$

37) $\frac{-6+i\sqrt{258}}{6}, \frac{-6-i\sqrt{258}}{6}$

38) $\frac{-6+i\sqrt{111}}{3}, \frac{-6-i\sqrt{111}}{3}$

39) $\frac{5+i\sqrt{130}}{5}, \frac{5-i\sqrt{130}}{5}$

40) $2, -4$

41) $\frac{-5+i\sqrt{87}}{2}, \frac{-5-i\sqrt{87}}{2}$

42) $\frac{-7+\sqrt{181}}{2}, \frac{-7-\sqrt{181}}{2}$

43) $\frac{3+i\sqrt{271}}{7}, \frac{3-i\sqrt{271}}{7}$

44) $\frac{-1+2i\sqrt{6}}{2}, \frac{-1-2i\sqrt{6}}{2}$

45) $\frac{7+i\sqrt{139}}{2}, \frac{7-i\sqrt{139}}{2}$

46) $\frac{5+3i\sqrt{7}}{2}, \frac{5-3i\sqrt{7}}{2}$

47) $\frac{12}{5}, -4$

48) $\frac{1+i\sqrt{511}}{4}, \frac{1-i\sqrt{511}}{4}$

49) $\frac{9+\sqrt{21}}{2}, \frac{9-\sqrt{21}}{2}$

50) $\frac{1+i\sqrt{163}}{2}, \frac{1-i163}{2}$

51) $\frac{-5+i\sqrt{415}}{8}, \frac{-5-i\sqrt{415}}{8}$

52) $\frac{11+i\sqrt{95}}{6}, \frac{11-i\sqrt{95}}{6}$

53) $\frac{5+i\sqrt{191}}{2}, \frac{5-i\sqrt{191}}{2}$

54) $8, 7$

55) $1, -\frac{5}{2}$

56) $3, -\frac{3}{2}$

9.4

Answers - Quadratic Formula

1) $\frac{i\sqrt{6}}{2}, -\frac{i\sqrt{6}}{2}$

2) $\frac{i\sqrt{6}}{3}, -\frac{i\sqrt{6}}{3}$

3) $2+\sqrt{5}, 2-\sqrt{5}$

4) $\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6}$

5) $\frac{\sqrt{6}}{2}, -\frac{\sqrt{6}}{2}$

6) $\frac{-1+i\sqrt{29}}{5}, \frac{-1-i\sqrt{29}}{5}$

7) $1, -\frac{1}{3}$

8) $\frac{1+\sqrt{31}}{2}, \frac{1-\sqrt{31}}{2}$

9) $3, -3$

10) $i\sqrt{2}, -i\sqrt{2}$

11) $3, 1$

12) $-1+i, -1-i$

13) $\frac{-3+i\sqrt{55}}{4}, \frac{-3-i\sqrt{55}}{4}$

14) $\frac{-3+i\sqrt{159}}{12}, \frac{-3-i\sqrt{159}}{12}$

15) $\frac{-3+\sqrt{141}}{6}, \frac{-3-\sqrt{141}}{6}$

16) $\sqrt{3}, -\sqrt{3}$

17) $\frac{-3+\sqrt{401}}{14}, \frac{-3-\sqrt{401}}{14}$

18) $\frac{-5+\sqrt{137}}{8}, \frac{-5-\sqrt{137}}{8}$

19) $2, -5$

20) $5, -9$

21) $\frac{-1+i\sqrt{3}}{2}, \frac{-1-i\sqrt{3}}{2}$

22) $3, -\frac{1}{3}$

23) $\frac{7}{2}, -7$

24) $\frac{-3+i\sqrt{3}}{3}, \frac{-3-i\sqrt{3}}{3}$

25) $\frac{7+3\sqrt{21}}{10}, \frac{7-3\sqrt{21}}{10}$

26) $\frac{-5+\sqrt{337}}{12}, \frac{-5-\sqrt{337}}{12}$

27) $\frac{-3+i\sqrt{247}}{16}, \frac{-3-i\sqrt{247}}{16}$

28) $\frac{3+\sqrt{33}}{6}, \frac{3-\sqrt{33}}{6}$

29) $-1, -\frac{3}{2}$

30) $2\sqrt{2}, -2\sqrt{2}$

31) $4, -4$

32) $2, -4$

33) $4, -9$

34) $\frac{2+3i\sqrt{5}}{7}, \frac{2-3i\sqrt{5}}{7}$

35) $6, -\frac{9}{2}$

36) $\frac{5+i\sqrt{143}}{14}, \frac{5-i\sqrt{143}}{14}$

37) $\frac{-3+\sqrt{345}}{14}, \frac{-3-\sqrt{345}}{14}$

38) $\frac{\sqrt{6}}{2}, -\frac{\sqrt{6}}{2}$

39) $\frac{\sqrt{26}}{2}, -\frac{\sqrt{26}}{2}$

40) $\frac{-1+\sqrt{141}}{10}, \frac{-1-\sqrt{141}}{10}$

9.5

Answers - Build Quadratics from Roots

NOTE: There are multiple answers for each problem. Try checking your answers because your answer may also be correct.

1) $x^2-7x+10=0$

2) $x^2-9x+18=0$

3) $x^2-22x+40=0$

4) $x^2-14x+13=0$

5) $x^2-8x+16=0$

6) $x^2-9x=0$

7) $x^2=0$

8) $x^2+7x+10=0$

9) $x^2-7x-44=0$

10) $x^2-2x-3=0$

11) $16x^2-16x+3=0$

12) $56x^2-75x+25=0$

13) $6x^2-5x+1=0$

14) $6x^2-7x+2=0$

15) $7x^2-31x+12=0$

16) $9x^2-20x+4=0$

17) $18x^2-9x-5=0$

18) $6x^2-7x-5=0$

19) $9x^2+53x-6=0$

20) $5x^2+2x=0$

21) $x^2-25=0$

22) $x^2-1=0$

23) $25x^2-1=0$

24) $x^2-7=0$

25) $x^2-11=0$

26) $x^2-12=0$

27) $16x^2-3=0$

28) $x^2+121=0$

29) $x^2+13=0$

30) $x^2+50=0$

31) $x^2-4x-2=0$

32) $x^2+6x+7=0$

33) $x^2-2x+10=0$

34) $x^2+4x+20=0$

35) $x^2-12x+39=0$

36) $x^2+18x+86=0$

37) $4x^2+4x-5=0$

38) $9x^2-12x+29=0$

39) $64x^2-96x+38=0$

40) $4x^2+8x+19=0$

9.6

Answers - Quadratic in Form

1) $\pm 1, \pm 2$

2) $\pm 2, \pm\sqrt{5}$

3) $\pm i, \pm 2\sqrt{2}$

4) $\pm 5, \pm 2$

5) $\pm 1, \pm 7$

6) $\pm 3, \pm 1$

7) $\pm 3, \pm 4$

8) $\pm 6, \pm 2$

9) $\pm 2, \pm 4$

10) $2, 3, -1 \pm i\sqrt{3}, \frac{-3 \pm 3i\sqrt{3}}{2}$

11) $-2, 3, 1 \pm i\sqrt{3}, \frac{-3 \pm i\sqrt{3}}{2}$

12) $\pm\sqrt{6}, \pm 2i$

13) $\frac{\pm 2i\sqrt{3}}{3}, \frac{\pm\sqrt{6}}{2}$

14) $\frac{1}{4}, -\frac{1}{3}$

15) $-125, 343$

16) $-\frac{5}{4}, \frac{1}{5}$

17) $1, -\frac{1}{2}, \frac{1 \pm i\sqrt{3}}{4}, \frac{-1 \pm i\sqrt{3}}{2}$

18) $\pm 2, \pm\sqrt{3}$

19) $\pm i, \pm\sqrt{3}$

20) $\pm i\sqrt{5}, \pm i\sqrt{2}$

21) $\pm\sqrt{2}, \pm\frac{\sqrt{2}}{2}$

22) $\pm i, \frac{\pm 6}{2}$

23) $\pm 1, \pm 2\sqrt{2}$

24) $2, \sqrt[3]{2}, -1 \pm i\sqrt{3}, \frac{-\sqrt[3]{2} \pm i\sqrt[6]{108}}{2}$

25) $1, \frac{1}{2}, \frac{-1 \pm i\sqrt{3}}{4}, \frac{-1 \pm i\sqrt{3}}{2}$

26) $\frac{1}{2}, -1, \frac{-1 \pm i\sqrt{3}}{4}, \frac{1 \pm i\sqrt{3}}{2}$

27) $\pm 1, \pm i, \pm 2, \pm 2i$

28) $6, 0$

29) $-(b+3), 7-b$

30) -4

31) $-4, 6$

32) $8, -1$

33) $-2, 10$

34) $2, -6$

35) $-1, 11$

36) $\frac{5}{2}, 0$

37) $4, -\frac{4}{3}$

38) $\pm\sqrt{6}, \pm\sqrt{2}$

39) $\pm 1, -\frac{1}{3}, \frac{5}{3}$

40) $0, \pm 1, -2$

41) $\frac{511}{3}, -\frac{1339}{24}$

42) $-3, \pm 2, 1$

43) $\pm 1, -3$

44) $-3, -1, \frac{3}{2}, -\frac{1}{2}$

45) $\pm 1, -\frac{1}{2}, \frac{3}{2}$

46) $1, 2, \frac{1}{3}, -\frac{2}{3}$

9.7

Answers - Rectangles

1) 6 m x 10 m
2) 5
3) 40 yd x 60 yd
4) 10 ft x 18 ft
5) 6 x 10
6) 20 ft x 35 ft
7) 6" x 6"
8) 6 yd x 7 yd
9) 4 ft x 12 ft
10) 1.54 in
11) 3 in
12) 10 ft
13) 1.5 yd
14) 6 m x 8 m
15) 7 x 9
16) 1 in
17) 10 rods
18) 2 in
19) 15 ft
20) 20 ft
21) 1.25 in
22) 23.16 ft
23) 17.5 ft
24) 25 ft
25) 3 ft
26) 1.145 in

9.8

Answers - Teamwork

1) 4 and 6
2) 6 hours
3) 2 and 3
4) 2.4
5) C = 4, J = 12
6) 1.28 days
7) $1\frac{1}{3}$ days
8) 12 min
9) 8 days
10) 15 days
11) 2 days
12) $4\frac{4}{9}$ days
13) 9 hours
14) 12 hours
15) 16 hours
16) $7\frac{1}{2}$ min
17) 15 hours
18) 18 min
19) $5\frac{1}{4}$ min
20) 3.6 hours
21) 24 min
22) 180 min or 3 hrs
23) Su = 6, Sa = 12
24) 3 hrs and 12 hrs
25) P = 7, S = $17\frac{1}{2}$
26) 15 and 22.5 min
27) A = 21, B = 15
28) 12 and 36 min

9.9

Answers - Simultaneous Product

1) $(2,36),(-18,-4)$
2) $(-9,-20),(-40,-\frac{9}{2})$
3) $(10,15),(-90,-\frac{5}{3})$
4) $(8,15),(-10,-12)$
5) $(5,9),(18,2.5)$
6) $(13,5),(-20,-\frac{13}{4})$
7) $(45,2),(-10,-9)$
8) $(16,3),(-6,-8)$
9) $(1,12),(-3,-4)$
10) $(20,3),(5,12)$
11) $(45,1),(-\frac{5}{3},-27)$
12) $(8,10),(-10,-8)$

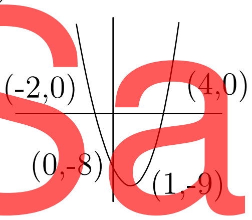
9.10

Answers - Revenue and Distance

1) 12
2) $4
3) 24
4) 55
5) 20
6) 30
7) 25 @ $18
8) 12 @ $6
9) 60 mph, 80 mph
10) 60, 80
11) 6 km/hr
12) 200 km/hr
13) 56, 76
14) 3.033 km/hr
15) 12 mph, 24 mph
16) 30 mph, 40 mph
17) r = 5
18) 36 mph
19) 45 mph
20) 40 mph, 60 mph
21) 20 mph
22) 4 mph

9.11

1)



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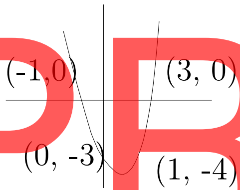

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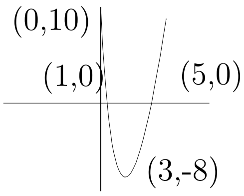

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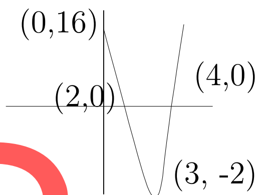

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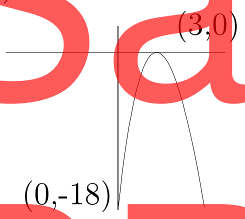

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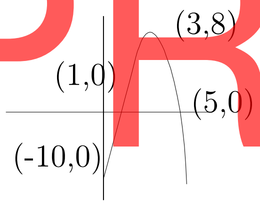

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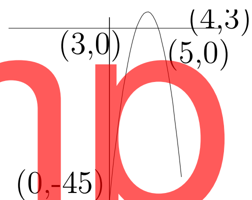

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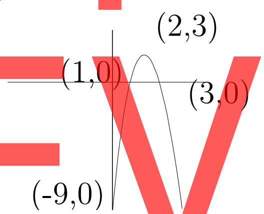

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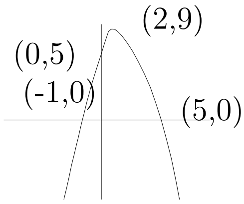

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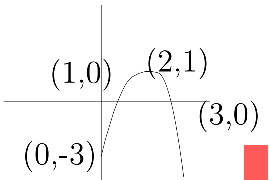

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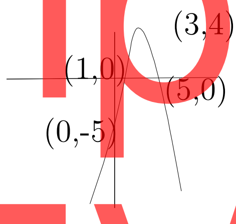

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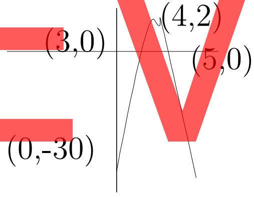

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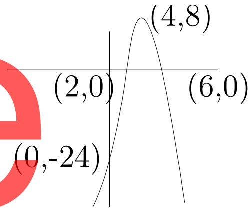

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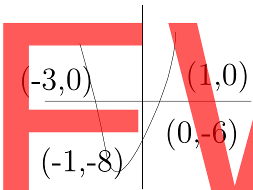

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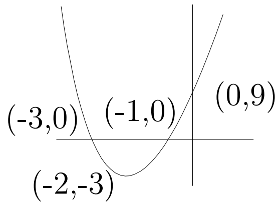

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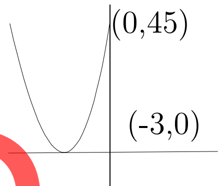

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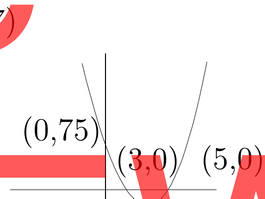

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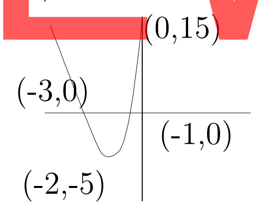

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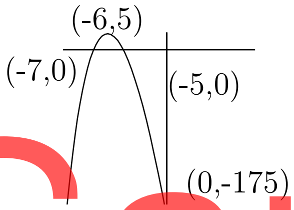

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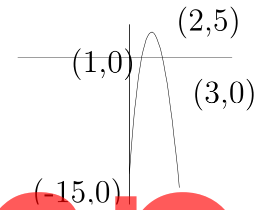

Answers - Chapter 10

10.1

Answers - Function Notation

1) a. yes b. yes c. no d. no e. yes f. no g. yes h. no

2) all real numbers

3) $x \leqslant \frac{5}{4}$

4) $t \neq 0$

5) all real numbers

6) all real numbers

7) $x \geqslant 16$

8) $x \neq -1, 4$

9) $x \geqslant 4, x \neq 5$

10) $x \neq \pm 5$

11) -4

12) $-\frac{3}{25}$

13) 2

14) 85

15) -7

16) 7

17) $-\frac{17}{9}$

18) -6

19) 13

20) 5

21) 11

22) -21

23) 1

24) -4

25) -21

26) 2

27) -60

28) -32

29) 2

30) $\frac{31}{32}$

31) $-64x^3+2$

32) $4n+10$

33) $-1+3x$

34) $-3 \cdot 2^{\frac{12+a}{4}}$

35) $2|-3n^2-1|+2$

36) $1+\frac{1}{16}x^2$

37) $3x+1$

38) t^4+t^2

39) 5^{-3-x}

40) $5^{\frac{-2+n}{2}}+1$

10.2

Answers - Operations on Functions

1) 82

2) 20

3) 46

4) 2

5) 5

6) -30

7) -3

8) 140

9) 1

10) -43