Six simple reasons why the mainstream derivative definition of calculus is flawed.

I had an email exchange with a student of mainstream mathematics who asked the following questions in response to some of mine in previous emails.

John Gabriel: If we write $\left|\frac{f(x+h)-f(x)}{h} - L\right| < \varepsilon$ then this means that the distance between $\frac{f(x+h)-f(x)}{h}$ and L becomes smaller as ε becomes smaller or rather that the distance $\left|\frac{f(x+h)-f(x)}{h} - L\right|$ can be made smaller than any given $\varepsilon > 0$ (*).

Student: This is what they call the rigorous definition, right?

John Gabriel: Essentially yes. It is normally stated as:

$$\forall\ \varepsilon,\delta > 0\ \exists\ \varepsilon > 0\ \text{ such that }\ 0 < |x - h| < \delta \rightarrow \left|\frac{f(x+h)-f(x)}{h} - L\right| < \varepsilon \quad \textbf{[IDE]}$$

However, it's not a well-formed definition, but a weakly stated property of this type of limit (derivative). It's unremarkable and problematic for several reasons.

1. The "definition" is *circular,* because in order to use it, one must know the value of L which is in fact the derivative $f'(x)$ in the case of $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$

and $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ is also known as the first principles method, that is, $f'(x)$ or L is used in its own definition:

$$\forall\ \varepsilon, \delta > 0\ \exists\ \varepsilon > 0 \text{ such that } 0 < |x - h| < \delta \to \left| \frac{f(x+h)-f(x)}{h} - f'(x) \right| < \varepsilon$$

2. There is no such thing as a ”real number”, that is, there is no valid construction of real number. None of Dedekind Cuts, Equivalent converging sequences or “Eudoxus reals” are valid.

By definition:

A number is a name given to a measure of a ratio.

3. The so-called definition is more of a *verifinition* (portmanteau of *verify* and *definition*) because it checks the guess L obtained from the flawed arithmetic in the first principles method where division by h is performed and after simplification taking the limit is equivalent to setting $h = 0$.

Any mainstream academic will vehemently claim that a limit is being taken. While this is partly true because the limit of the sequence of finite differences $\frac{f(x+h_i)-f(x)}{h_i}$ is the derivative, it is however equivalent to setting h=0, something they deny.

Proof that taking the limit and setting h=0 is the same:

Given any f(x), the finite difference quotient $\frac{f(x+h)-f(x)}{h}$ evaluates to f'(x) + Q(x,h) where Q(x,h) is the difference in slope between the tangent line and the non-parallel secant line. Q(x,h) is equal to 0 iff f(x) is a linear function.

1. **Taking the limit of the finite differences gives f'(x)**
2. **Discarding Q(x,h) is the same as setting Q(x,h) = 0 because all its terms have a factor of h.**
3. **Setting Q(x,h) = 0 is equivalent to setting h=0. Since taking the limit and setting h=0 produce the same result, it follows that they are equivalent and we are done since h=0 <=> taking limit as h tends to 0. Q.E.D.**

4. **The limit L itself is not contained in the sequence of finite differences:**

$$\frac{f(x+h_0)-f(x)}{h_0} \; ; \; \frac{f(x+h_1)-f(x)}{h_1} \; ; \; \frac{f(x+h_2)-f(x)}{h_2} \; ; \; \ldots \; ; \; \frac{f(x+h_n)-f(x)}{h_n} \; ; \; \ldots$$

There simply is not any finite difference of the form $\frac{f(x+h_i)-f(x)}{h_i}$ which will ever be equal to the limit in terms of the function f, only in terms of the tangent line to f at x. My New Calculus solves this problem rigorously for the first time in human history.

5. **In order for the verifinition to work, it must be possible to have a hole (*) at the point of tangency.**

This is mandated by the requirement that both ε and δ in the inequality they call the "definition" must be greater than ZERO, namely [IDE].

6. My historic geometric theorem reveals the identity $\frac{f(x+h)-f(x)}{h} = f'(x) + Q(x,h)$ which exposes a fatal flaw in the reasoning of mainstream math academics. If one takes the limit of both sides of the above identity, it turns out that only one of the constants remains the same, namely $f'(x)$.

Student: How is this definition significant?

It's a testament to the utter failure, incompetence and grand stupidity of mainstream academics since Newton to realise a rigorous formulation of calculus. There is nothing rigorous about $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ and there never was.

Student: There is nothing one can do with it, because there is no mathematical operation known as 'becomes smaller'.

Right. Mathematically, "becomes smaller", "tends to infinity", etc, are all obscure phrases that lack any of the characteristic known as rigour.

Student: It is understood what is signified and needs to be done whenever one sees any of these operators +, − , ÷ or ×. However, there is nothing one can do with *lim h* → *c* because *h* 'approaches' *c* or the distance between *c* and *h* becoming “smaller” is not a mathematical operation. Hence, rigour is nowhere to be observed.

Indeed. However, to mainstream academics, "becomes smaller" means "*c* closer *h* to” but |*c*-*h*| can never actually be 0. For this reason the hole feature (*) previously mentioned is required.

Student: Is this reasoning correct?

Your basic reasoning is correct.

Student: And, is this essentially the reason behind the statement " *There is no valid systematic way of finding a derivative in mainstream calculus* "?

True for all the reasons I stated above and more. Another significant reason is that if one takes the limit of constants (as mentioned earlier), then the constants are themselves the limits - they cannot be something else, for even by mainstream decree, the limit of a constant is always the limit itself. From my historic geometric theorem we see that the constant slope of the non-parallel secant line $\frac{f(x+h)-f(x)}{h}$ becomes the slope of the tangent line $f'(x)$ on the left hand side of my identity

$$\frac{f(x+h)-f(x)}{h} = f'(x) + Q(x,h).$$

Only $f'(x)$ stays constant on the right hand side but the limit of the difference constant $Q(x,h)$ is ZERO and somehow $\frac{f(x+h)-f(x)}{h}$ magically morphs into the derivative $f'(x)$ when it is clearly impossible for any of the finite differences $\frac{f(x+h_i)-f(x)}{h_i}$ to represent $f'(x)$, for certain none of the “infinitely many” in the sequence:

$$\frac{f(x+h_0)-f(x)}{h_0} ; \frac{f(x+h_1)-f(x)}{h_1} ; \frac{f(x+h_2)-f(x)}{h_2} ; \ldots ; \frac{f(x+h_0)-f(x)}{h_n} ; \ldots$$

Therefore, the mainstream formulation is a fraud whether sincere or otherwise understood.

The email correspondence ended with this response from the student:

Thanks.
There are millions of STEM students/educators/professionals who deal with calculus on a daily basis. Therefore, it is quite noteworthy or fascinating that calculus is not questioned more. The reason for this is that most people are simply not seekers of truth. They do not care. For instance, students, even in university, only care about marks. Similarly, most educators or professionals only care about getting their salary.

Another reason is that people assume that they themselves are stupid. If they do not understand limits that means they are dumb, while smart people do easily understand.

Lastly, people probably do not have the courage to question orthodox doctrine.
"Who am I to question question established theory? Who am I to contradict Cauchy, Cantor etc." the thinking goes.

Still, I find it remarkable that there are not a handful of individuals at almost every university who question mainstream mathematics.
Even if they do not do so openly, one would expect the internet to be full of criticism regarding mainstream mathematics. But alas, the opposite is true.